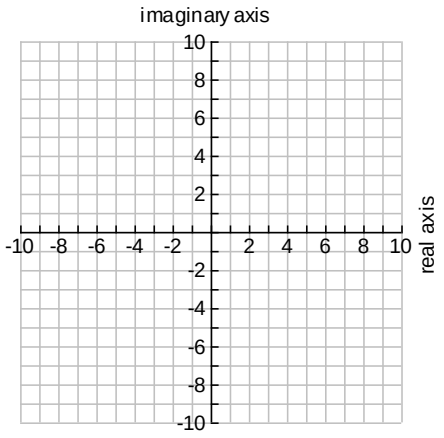
SM 3H

Date: **Section:**

Objective:

The complex number $a+bi$ can be thought of as an ordered pair (a,b). We graph it on the **complex plane** where the horizontal axis is called the _____ **axis** and the vertical axis is called the _____ **axis.**



Absolute Value or Modulus: $|a+bi| =$ _____. (The distance between the number and the origin on the complex plane.)

Examples: Graph each complex number and find its absolute value.

a) $5-i$ b) $-6+2i$

Trigonometric Form of a Complex Number

If $z=a+bi$ is a complex number, then the trigonometric form of z is

$z =$ _____, sometimes abbreviated $z = r\operatorname{cis}\theta,$

where r is called the _____ and θ is called the _____**,** defined as the angle in standard position whose terminal side contains the point (a,b).

$r =$ _____

$a =$ _____ and $b =$ _____.

We usually use the smallest possible nonnegative angle for θ.

Examples: Write each complex number in trigonometric form. Express θ in degrees.

a) $-2\sqrt{3}+2i$ b) $5-4i$

Example: Write the complex number $12\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ in the form $a+bi$.

Product and Quotient of Complex Numbers

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$$z_1 z_2 = \_\_\_\_\_$$

$$\frac{z_1}{z_2} = \_\_\_\_\_$$

Examples: Find the product and quotient using trigonometric form.

$$z_1 = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right), \quad z_2 = 8\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

a) Find $z_1 z_2$

b) Find $\frac{z_1}{z_2}$

Find the quotient for each pair of complex numbers, using trigonometric form. Write the answer in standard form for complex numbers.

a) $z_1 = 3 + 4i, \quad z_2 = -5 + 2i$

Complex Conjugates

The conjugate of $r(\cos(\theta) + i\sin(\theta))$ is _____

A complex number times its conjugate equals _____.

Proof: $r(\cos\theta + i\sin\theta)\cdot r(\cos(-\theta) + i\sin(-\theta))$

$= r^2(\cos(\theta-\theta) + i\sin(\theta-\theta))$

$= r^2(\cos 0 + i\sin 0)$

$= r^2(1 + 0i) = r^2$

Example: Find the product of the following and its conjugate: $6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.