SM 3H

Date: 5/13/24 **Section:** 9.5

Objective: I can find vector quantities and do vector operations.

Vocabulary

1. Scalar quantities: linear measurments
   ex. height, distance, width, circumference, perimeter
2. Vector quantities: linear measurements with direction
   ex. velocity, force, acceleration
3. Vector direction and magnitude of line segment
   notation or $\mathbf{v}$
4. Magnitude: distance of vector

Notations: $|\mathbf{v}|$ or $\vec{v}$ or $||\mathbf{v}||$ or $||\vec{v}||$

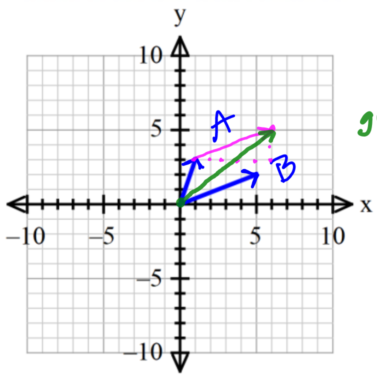
5. Direction: angle of direction

   Notation: θ

6. Equal Vectors – same distance & direction but start in a dif spot
7. Zero Vector: point, no direction + no magnitude
8. Scalar Multiplication: mult by GCF

Vector Addition: the vector made when putting 2 vectors tail to head

Resultant vector: r = the new vector

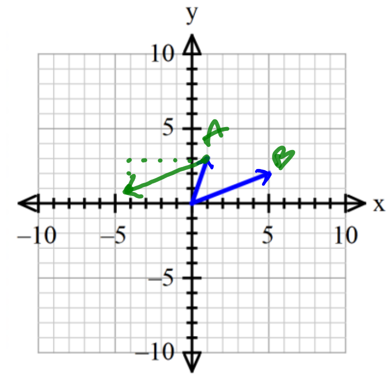


Steps for addition and subtraction (A + B or A – B)

green = resultant vector

$\langle 6,5 \rangle$

$\langle -4,1 \rangle$



A + B =

$\langle 1,3 \rangle + \langle 5,2 \rangle = \langle 6,5 \rangle$

add x-coor add y-coor

A – B =

$\langle 1,3 \rangle - \langle 5,2 \rangle = \langle -4,1 \rangle$

Steps for scalar multiplication (kA)

2A =

3B =

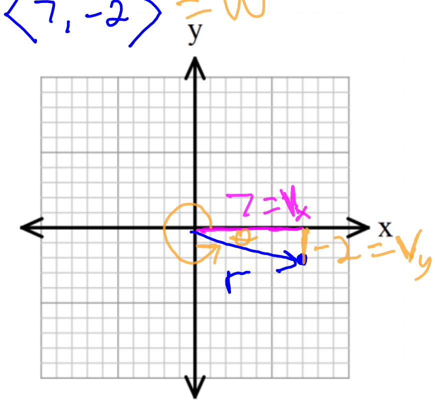
$2\langle 1,3 \rangle = \langle 2,6 \rangle$

VECTORS have 2 parts

Horizontal component: V_x

Vertical component: V_y

ex. $\langle 7,-2 \rangle = W$



$r = \sqrt{7^2 + 2^2} = \sqrt{53}$

Position Vector: resultant vector

Direction Angle: – find angle, check Quad

$\theta = \tan^{-1}\left(\frac{-2}{7}\right) \approx -15.95°$

$344.05°$

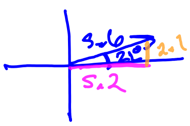
$\cos\theta = \dfrac{V_x}{r} = \dfrac{adj}{hyp}$ $\sin\theta = \dfrac{V_y}{r} = \dfrac{opp}{hyp}$

r: resultant $r = \sqrt{V_x^2 + V_y^2}$

OR

$|A_x| = r\cos\theta$ $|A_y| = r\sin\theta$

Example: Find the magnitude of $\boldsymbol{V_x}$ and $\boldsymbol{V_y}$ if $r = |\boldsymbol{V}| = 5.6$ and $\theta = 22°$



$V_x = 5.6\cos 22° \approx 5.2$

$V_y = 5.6\sin 22° \approx 2.1$

Example: Write the above vector in component form. –ordered pair

$\langle 5.2,\ 2.1\rangle$

To find magnitude:

To find direction angle:

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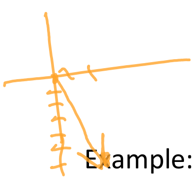
Example: Find the magnitude and direction of the vector $\boldsymbol{w} = \langle 2,\ -6\rangle$.



$r = \sqrt{2^2 + 6^2} = 2\sqrt{10}$

$\theta = \tan^{-1}\left(\frac{-6}{2}\right) \approx -71.6°$

$360°$

$\theta \approx 288.4°$

Example: Find the following operations with the vectors $\boldsymbol{w} = \langle -1,\ -3\rangle$ and $\boldsymbol{v} = \langle 3,\ -4\rangle$.

1. $\boldsymbol{w} + \boldsymbol{v}$ $= \langle 2, -7\rangle$

2. $\boldsymbol{w} - \boldsymbol{v}$ $= \langle -4, 1\rangle$

3. $-8\boldsymbol{v}$ $= \langle -24, 32\rangle$

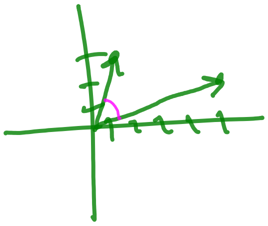
4. $3\boldsymbol{w} + 4\boldsymbol{v}$ $= \langle -3, -9\rangle + \langle 12, -16\rangle = \langle 9, -25\rangle$

Dot Product: $w_x v_x + w_y v_y$

5. $\boldsymbol{w} \cdot \boldsymbol{v}$ $-3 + 12 = 9$

To find the angle between vectors: $\cos^{-1}\left(\frac{w \cdot v}{|w| \cdot |v|}\right) = \theta$

Example: Find the smallest angle between the vectors $\boldsymbol{w} = \langle 1,\ 3\rangle$ and $\boldsymbol{v} = \langle 5,\ 2\rangle$.



$w \cdot v = 5 + 6 = 11$

$|w| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$|v| = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\cos^{-1}\left(\underline{\qquad\qquad}\right)$

$\cos^{-1}\left(\frac{11}{\sqrt{10} \cdot \sqrt{29}}\right) = \theta$

$\theta \approx 49.8°$

<u>Vocabulary:</u>

Parallel: never cross – same slope

Perpendicular or orthogonal: 90° meet – slopes are opp & reciprocal

To find parallel vectors: if have GCF

$\langle v_x, v_y\rangle = k\langle w_x, w_y\rangle$

To find orthogonal vectors: $w \cdot v = 0$ dot product $= 0$

Examples: Are the following vectors parallel or orthogonal or neither?

1. $\boldsymbol{w} = \langle -2,\ 3\rangle$ and $\boldsymbol{v} = \langle 6,\ 4\rangle$

$\langle -2, 3\rangle \rightarrow 2\langle 3, 2\rangle$ not parallel

$-12 + 12 = 0$ perp

2. $\boldsymbol{w} = \langle 2, -5\rangle$ and $\boldsymbol{v} = \langle -4,\ 10\rangle$

$\langle 2, -5\rangle \rightarrow -2\langle +2, -5\rangle$

parallel

To find unit vectors in linear combination:

The unit vectors are $v_x = \langle 1, 0\rangle\ i$ and $v_y = \langle 0, 1\rangle\ j$

Example: Write the following vector in linear combination.

$\boldsymbol{w} = \langle -6,\ 10\rangle$

$-6i + 10j$