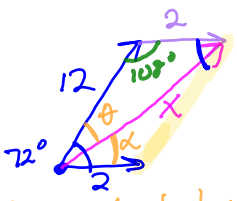
SM 3H

Date: 5/15/24 **Section:** 9.6

Objective: I can use vectors to solve stories.

Example: Forces of 2 lb and 12 lb are acting at an angle of 72° to each other. Find the magnitude of the resultant force and the angle between the resultant and each force.

X = resultant force



$X = \sqrt{2^2 + 12^2 - 2(2)(12)\cos 108°}$

$X \approx 12.76$ lb

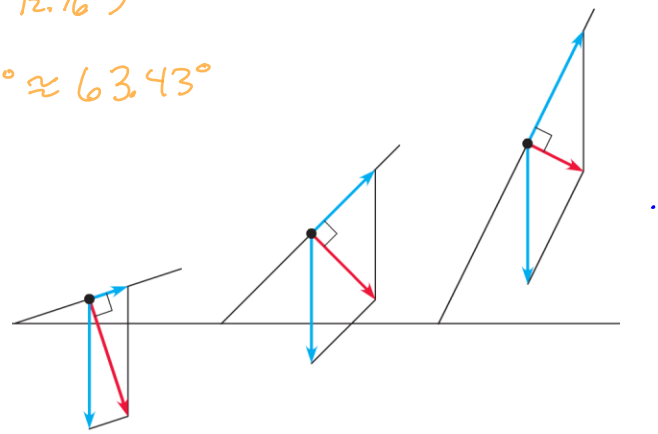
$\frac{\sin\theta}{2} = \frac{\sin 108°}{12.76}$ $\sin^{-1}\left(\frac{2\sin 108°}{12.76}\right) = \theta \approx 8.57°$

θ = angle between 12 & R.F.

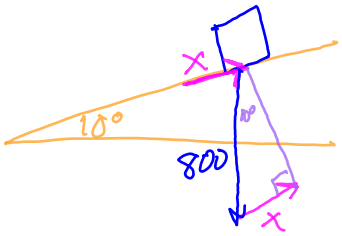
α = angle between 2 & R.F.

$\alpha = 72° - 8.57° \approx 63.43°$

Inclined Plane Problems: The weight of an object is always modeled as a vertical vector and the force required to move the object is modeled as a vector parallel to the inclined plane. Its length increases as the incline increases. The resultant of these two forces is a vector perpendicular to the inclined plane. It is what a bathroom scale would read if trapped between the object and the plane.



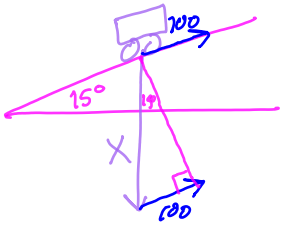
Example: Find the amount of force required to push an 800-pound block of ice up a ramp that is inclined 10°. = x



$\sin 10° = \frac{x}{800}$ 800

$x \approx 138.91$ lb

Example: A landscaper uses 100 pounds of force to pull a cart full of rocks up a driveway that is inclined 15°. What is the weight of the cart? = x

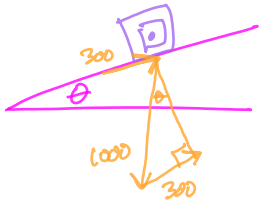


$\sin 15° = \frac{100}{x}$

$100\left(\frac{1}{\sin 15°} = \frac{x}{100}\right)$

$x \approx 386.37$ lb.

Example: If 300 pounds of force is required to push a 1000-pound safe up a ramp, then what is the angle of inclination of the ramp? = θ

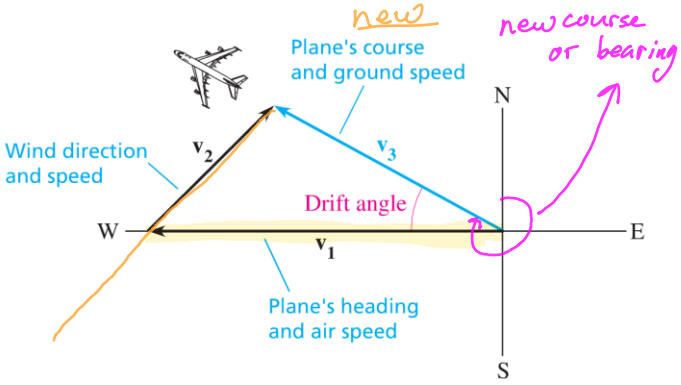


$\sin\theta = \frac{300}{1000}$

$\sin^{-1}\left(\frac{3}{10}\right)$

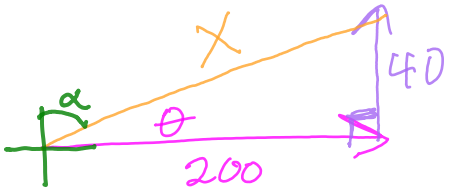
$\theta \approx 17.46°$

Navigation Problems: Wind affects the speed and direction of a plane. The **heading** and **air speed** are the direction and speed of the plane before wind is taken into account. The **course** and **ground speed** are the direction and speed of the plane with wind taken into account. The angle between the heading and the course is the **drift angle**.



Example: An airplane is headed due east with an air speed of 200 mph. The wind is out of the south (bearing 0°) at 40 mph. Find the drift angle, the ground speed, and the course of the airplane.

θ x α



$x = \sqrt{40^2 + 200^2} \approx 203.96 \text{ mph}$

$\theta = \tan^{-1}\left(\frac{40}{200}\right) \approx 11.31°$

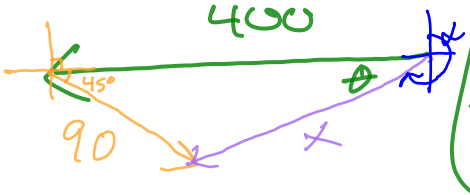
$\alpha = 90° - 11.31° \approx 78.69°$

Example: An airplane is headed due west with an air speed of 400 mph. The wind is out of the northwest (bearing 135°) at 90 mph. Find the drift angle, the ground speed and the course of the airplane.

θ x α

$x = \sqrt{400^2 + 90^2 - 2(400)(90)\cos 45°}$

$x \approx 342.33 \text{ mph}$



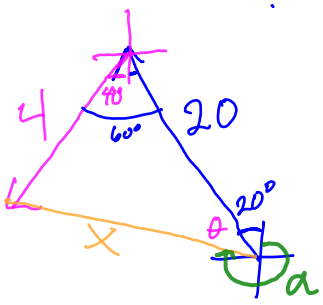
$\left(\frac{\sin\theta}{90} = \frac{\sin 45°}{342.33}\right) 90$

$\sin^{-1}\left(\frac{90 \sin 45°}{342.33}\right) = \theta \approx 10.71°$

$\alpha = 270° - 10.71° \approx 259.29°$

Example: A jet is headed northwest with an air speed of 500 mph. The wind is 100 mph with a bearing of 200°. Find the drift angle, the ground speed, and the course of the jet.

Example: A boat is traveling at 20 mph with a bearing of N20°W. The current is moving at 4mph with a bearing of S40°W. Find the boat's true speed, drift angle, and course.



X → θ → α

$$X = \sqrt{4^2 + 20^2 - 2(4)(20)\cos 60^\circ}$$

$$X \approx 18.33 \text{ mph}$$

$$\frac{\sin\theta}{4} = \frac{\sin 60^\circ}{18.33} \qquad \sin^{-1}\left(\frac{4\sin 60^\circ}{18.33}\right) = \theta \approx 10.89^\circ$$

$$\alpha = 360^\circ - (20^\circ + 10.89^\circ) \approx 329.11^\circ$$