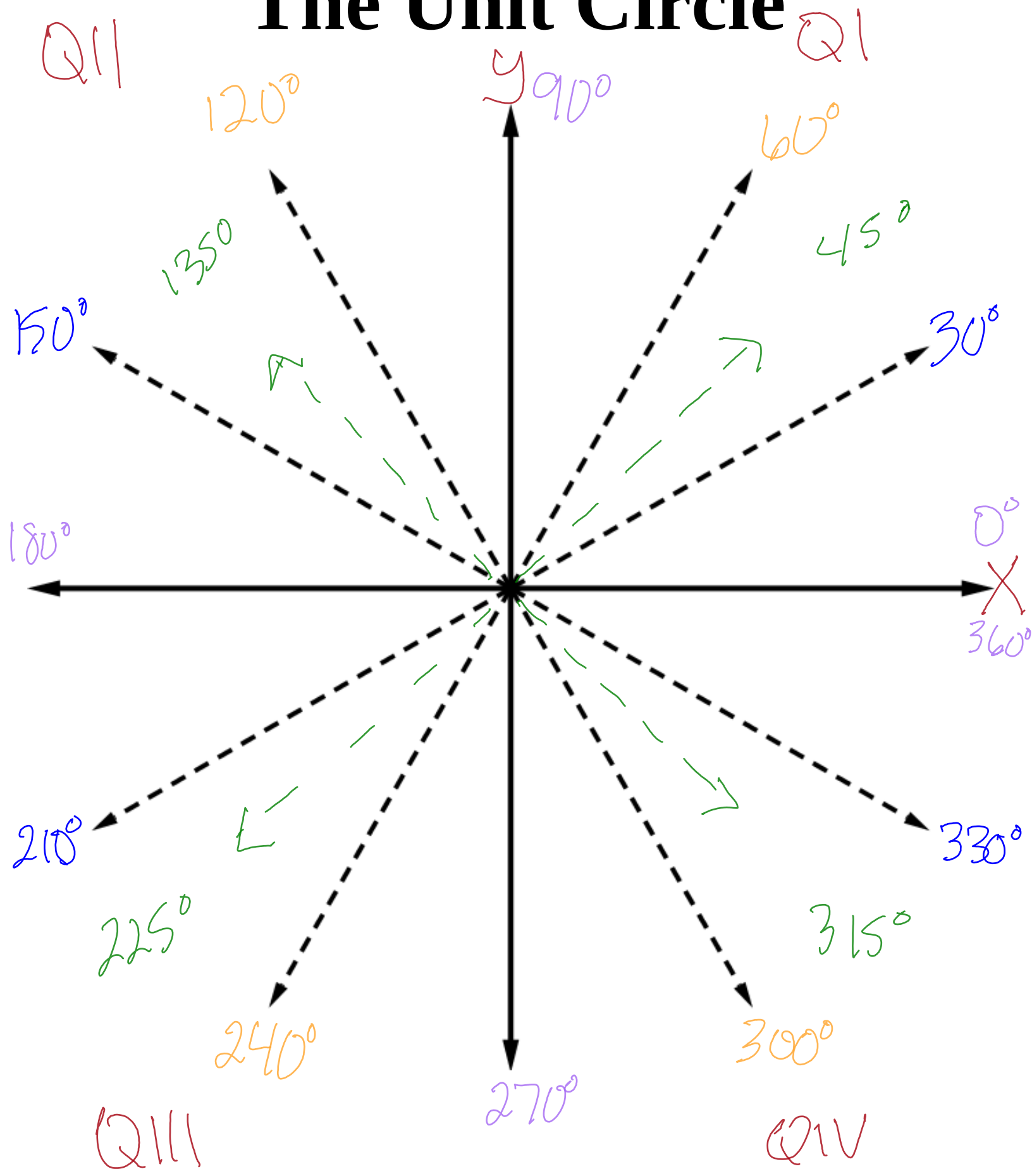
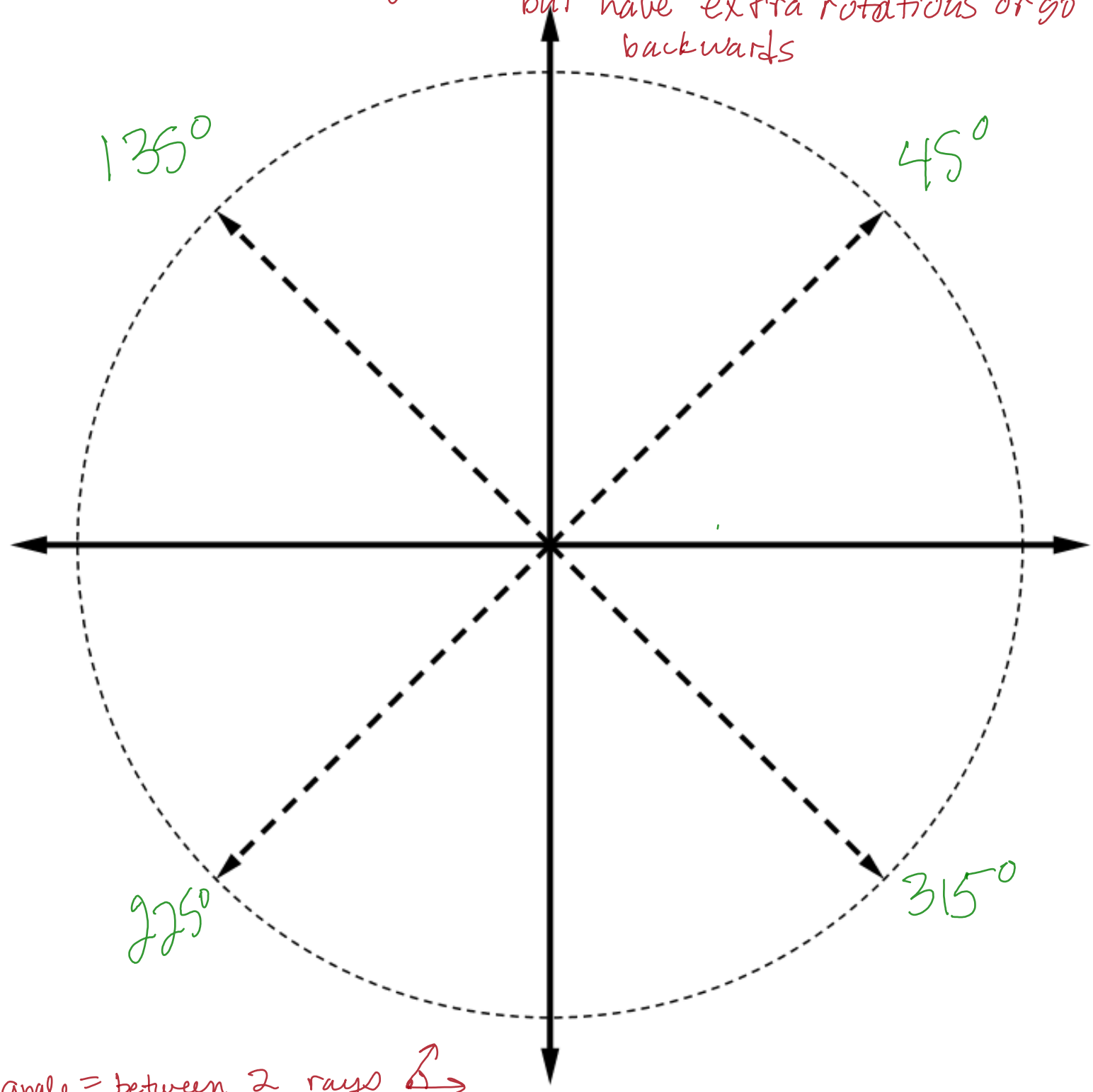
The Unit Circle



coterminal angles - start & stop in same place, but have extra rotations or go backwards



angle = between 2 rays

standard position = put angle on x-y-plane

initial side = side you start angle = on positive x-axis

terminal side = side where angle stops

positive angles go counter clockwise

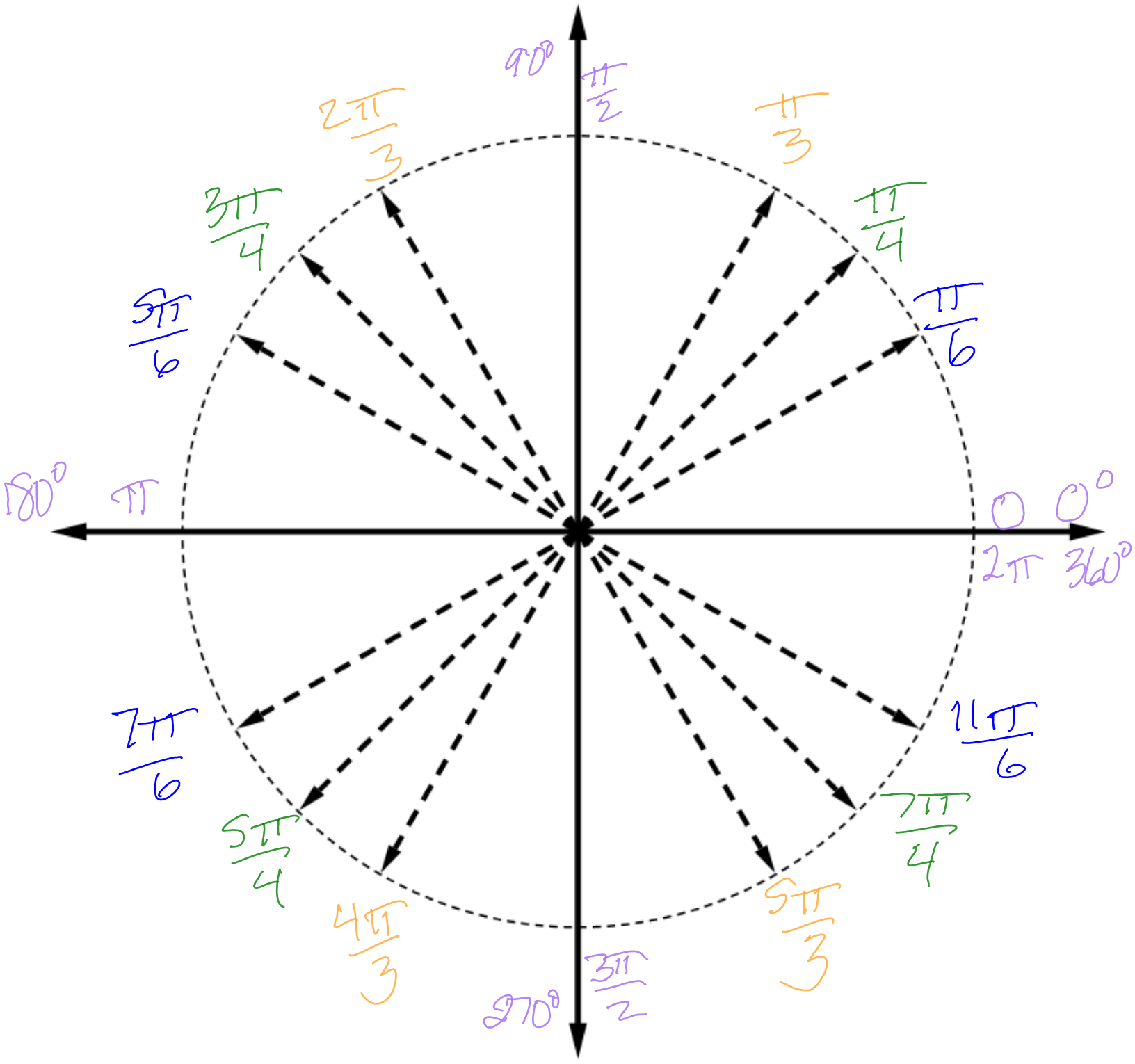
negative angles go clockwise

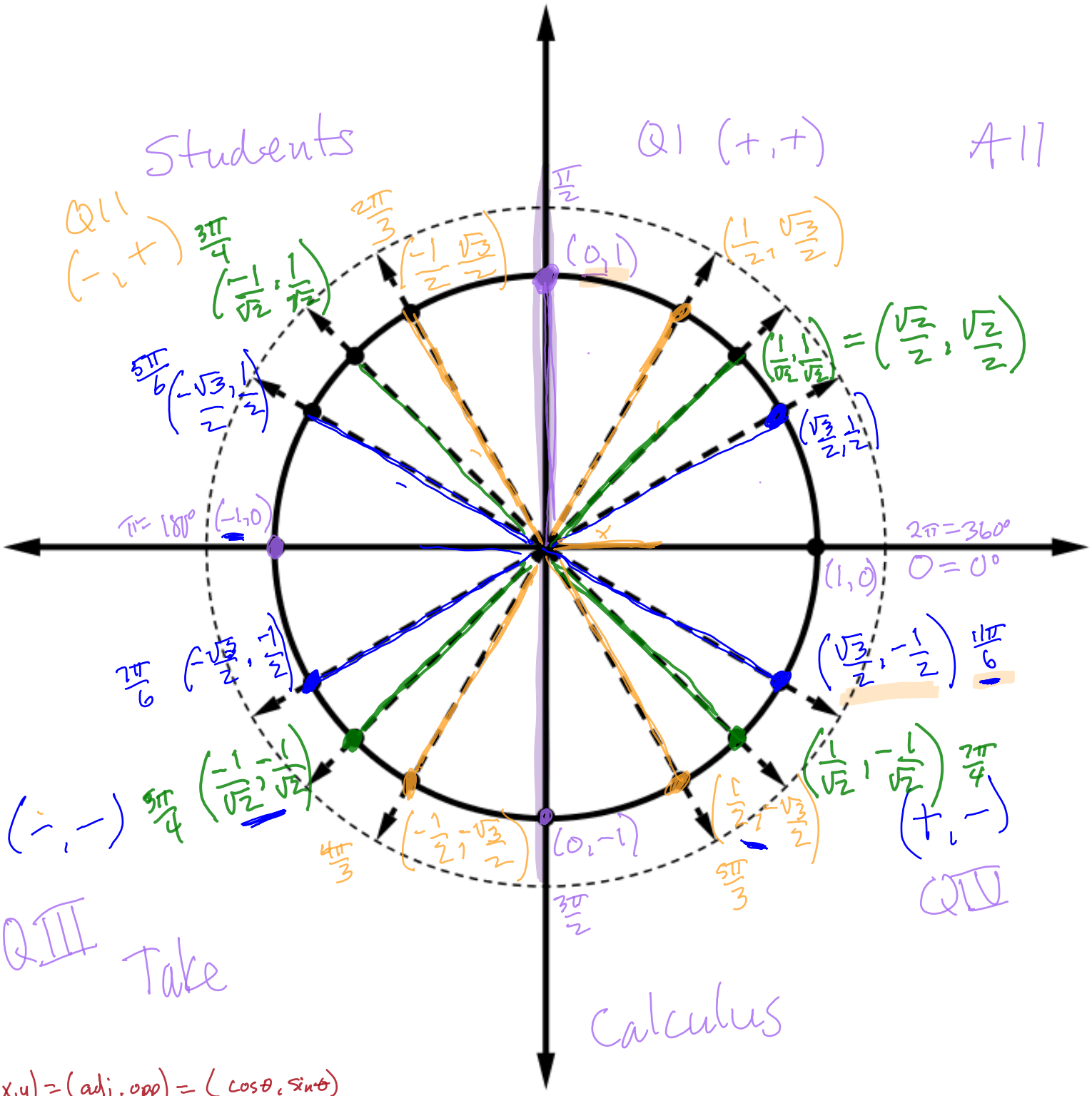
between 0° & 180° There are 6 30° angles

between 0° & 180° There are 3 60° angles

between 0° & 180° There are 2 90° angles

between 0° & 180° There are 4 45° angles


90°
π/2
2π/3
π/3
3π/4
π/4
5π/6
π/6
180°
π
0
0°
2π
360°
7π/6
11π/6
5π/4
7π/4
4π/3
5π/3
3π/2
270°



$(x,y) = (adj, opp) = (\cos\theta, \sin\theta)$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$

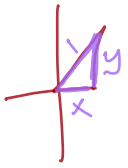
$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$

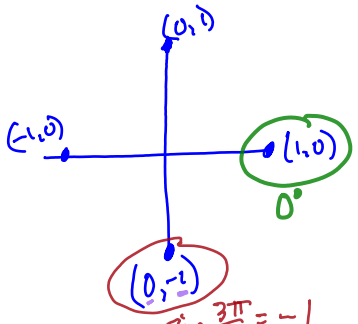
$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = \frac{hyp}{opp} = \frac{1}{y}$

$\sec\theta = \frac{hyp}{adj} = \frac{1}{x}$

$\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$





$\sin 0° = \frac{0}{1}$ $\quad \csc 0° = \frac{1}{0} = \text{undef}$

$\cos 0° = \frac{1}{1}$ $\quad \sec 0° = 1$

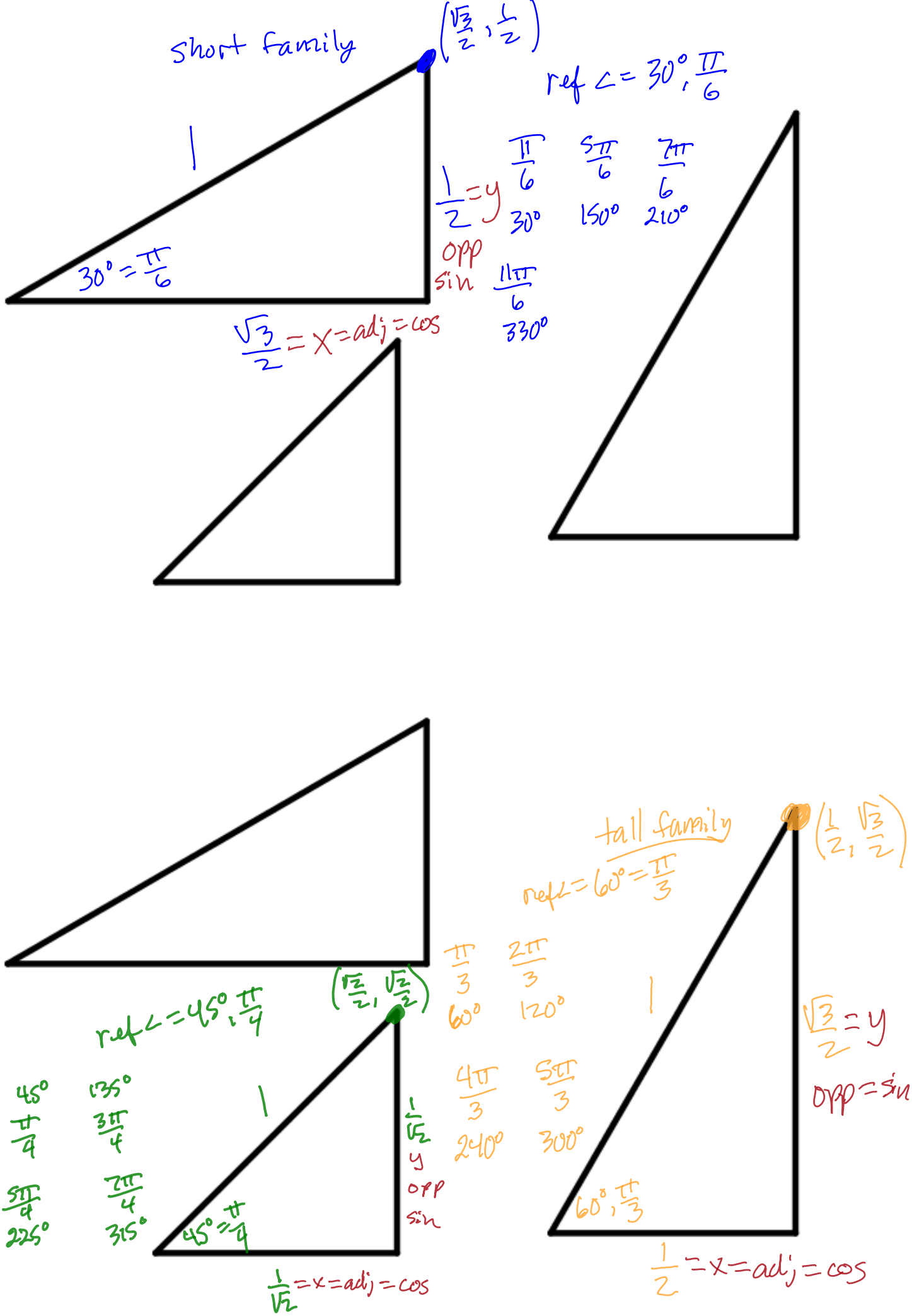
$\tan 0° = \frac{0}{1} = 0$ $\quad \cot 0° = \frac{1}{0} = \text{undef}$

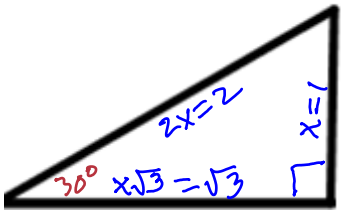
☆ 0 and undefined are reciprocals

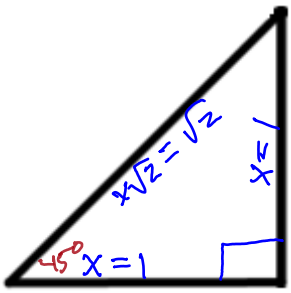
$\sin\frac{3\pi}{2} = -1$ $\quad \csc\frac{3\pi}{2} = -1$

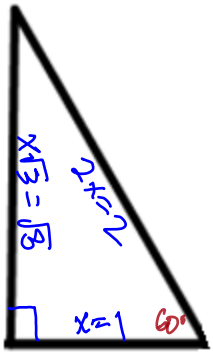
$\cos\frac{3\pi}{2} = 0$ $\quad \sec\frac{3\pi}{2} = \text{undef}$

$\tan\frac{3\pi}{2} = \text{undef}$ $\quad \cot\frac{3\pi}{2} = 0$


short family
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
ref $\angle = 30°, \frac{\pi}{6}$
1
$\frac{1}{2} = y$
opp
sin
$30° = \frac{\pi}{6}$
$\frac{\sqrt{3}}{2} = x = adj = \cos$
$\frac{\pi}{6}$ 30°
$\frac{5\pi}{6}$ 150°
$\frac{7\pi}{6}$ 210°
$\frac{11\pi}{6}$ 330°
tall family
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
ref $\angle = 60° = \frac{\pi}{3}$
$\frac{\pi}{3}$ 60°
$\frac{2\pi}{3}$ 120°
$\frac{4\pi}{3}$ 240°
$\frac{5\pi}{3}$ 300°
1
$\frac{\sqrt{3}}{2} = y$
opp = sin
$60°, \frac{\pi}{3}$
$\frac{1}{2} = x = adj = \cos$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
ref $\angle = 45°, \frac{\pi}{4}$
45° $\frac{\pi}{4}$
135° $\frac{3\pi}{4}$
225° $\frac{5\pi}{4}$
315° $\frac{7\pi}{4}$
1
$\frac{1}{\sqrt{2}}$
y
opp
sin
$45° = \frac{\pi}{4}$
$\frac{1}{\sqrt{2}} = x = adj = \cos$


2x=2
x=1
30°
x√3=√3


x√2=√2
x=1
45°
x=1


x√3=√3
2x=2
x=1
60°