**Definitions of Trigonometric Functions:**

If $(x, y)$ is a point on the terminal side of an angle $\alpha$ in standard position and $r=\sqrt{x^2+y^2}$, then

$$\sin\alpha=\frac{y}{r},\quad \cos\alpha=\frac{x}{r},\quad \tan\alpha=\frac{y}{x},\quad \csc\alpha=\frac{r}{y},\quad \sec\alpha=\frac{r}{x},\quad \cot\alpha=\frac{x}{y}$$

**Reciprocal Functions:**

$$\sin x=\frac{1}{\csc x}\quad \cos x=\frac{1}{\sec x}\quad \tan x=\frac{1}{\cot x}\quad \csc x=\frac{1}{\sin x}\quad \sec x=\frac{1}{\cos x}\quad \cot x=\frac{1}{\tan x}$$

**Tangent and Cotangent in Terms of Sine and Cosine:**

$$\tan x=\frac{\sin x}{\cos x}\quad \cot x=\frac{\cos x}{\sin x}$$

**Pythagorean Identities:**

$$\sin^2 x+\cos^2 x=1$$
$$\tan^2 x+1=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$

**Odd/Even Identities:**

$$\sin(-x)=-\sin x\quad \csc(-x)=-\csc x$$
$$\tan(-x)=-\tan x\quad \cot(-x)=-\cot x$$
$$\cos(-x)=\cos x\quad \sec(-x)=\sec x$$

**Cofunction Identities:**

$$\sin\left(\tfrac{\pi}{2}-x\right)=\cos x\quad \cos\left(\tfrac{\pi}{2}-x\right)=\sin x\quad \sin(90^\circ-x)=\cos x\quad \cos(90^\circ-x)=\sin x$$
$$\tan\left(\tfrac{\pi}{2}-x\right)=\cot x\quad \cot\left(\tfrac{\pi}{2}-x\right)=\tan x\quad \tan(90^\circ-x)=\cot x\quad \cot(90^\circ-x)=\tan x$$
$$\sec\left(\tfrac{\pi}{2}-x\right)=\csc x\quad \csc\left(\tfrac{\pi}{2}-x\right)=\sec x\quad \sec(90^\circ-x)=\csc x\quad \csc(90^\circ-x)=\sec x$$

**Sum and Difference Identities:**

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

**Double-Angle Identities:**

$$\sin(2x)=2\sin x\cos x$$
$$\cos(2x)=\cos^2 x-\sin^2 x$$
$$\cos(2x)=1-2\sin^2 x$$
$$\cos(2x)=2\cos^2 x-1$$
$$\tan(2x)=\frac{2\tan x}{1-\tan^2 x}$$

**Half-Angle Identities:**

$$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$$
$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$$
$$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$

**Key Points:**

**$y = \sin x$**

| $x$ | $0$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | $0$ | $1$ | $0$ | $-1$ | $0$ |

**$y = \cos x$**

| $x$ | $0$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | $1$ | $0$ | $-1$ | $0$ | $1$ |

**$y = \csc x$**

| $x$ | $0$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | undef | $1$ | undef | $-1$ | undef |

**$y = \sec x$**

| $x$ | $0$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | $1$ | undef | $-1$ | undef | $1$ |

**$y = \tan x$**

| $x$ | $-\pi/2$ | $-\pi/4$ | $0$ | $\pi/4$ | $\pi/2$ |
|---|---|---|---|---|---|
| $y$ | undef | $-1$ | $0$ | $1$ | undef |

**$y = \cot x$**

| $x$ | $0$ | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$ |
|---|---|---|---|---|---|
| $y$ | undef | $1$ | $0$ | $-1$ | undef |

**To Graph $y = a\,f\left[b(x-c)\right]+d$:**

1. Start with the key points of $y = f(x)$.
2. Multiply the $y$-coordinates by $a$ and add $d$.
3. Divide the $x$-coordinates by $b$ and add $c$.
4. Plot the new points and connect to form the graph.

**Amplitude:** $\text{amplitude} = |a|$ (for sin or cos)

**Period:** $\text{period} = \dfrac{2\pi}{b}$ (for sin, cos, csc, and sec) or

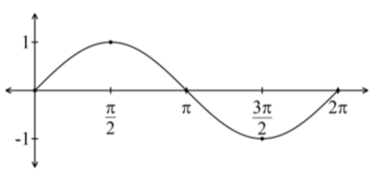
$\text{period} = \dfrac{\pi}{b}$ (for tan and cot)

**Frequency:** $\text{frequency} = \dfrac{1}{\text{period}}$

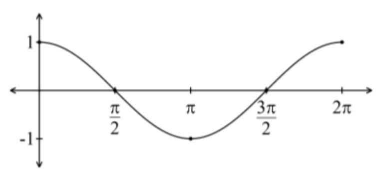
**Phase Shift:** $c$ (positive if shifted right, negative if shifted left)

**Vertical Shift:** $d$ (positive if shifted up, negative if shifted down)

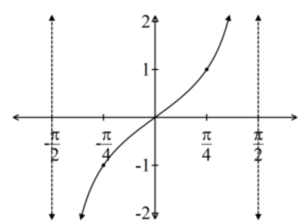
**Asymptotes:** $x = \text{first non-negative asymptote} + (\text{distance between asymptotes}) \cdot k$



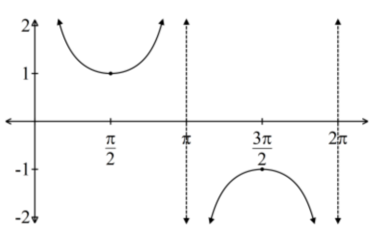
$y = \sin x$



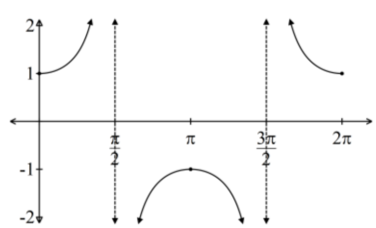
$y = \cos x$



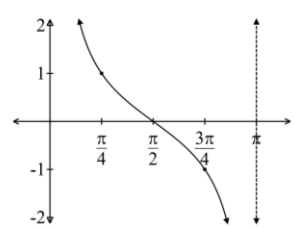
$y = \tan x$



$y = \csc x$



$y = \sec x$



$y = \cot x$

**Arc Length:**

$$s = \alpha r \ (\alpha \text{ must be in radians}) \text{ or } s = \frac{\alpha}{360^\circ} \cdot \text{circumference} \ (\alpha \text{ in degrees})$$

**Sector Area:**

$$A = \frac{\alpha r^2}{2} \ (\alpha \text{ must be in radians}) \text{ or } A = \frac{\alpha}{360^\circ} \cdot \text{area of circle} \ (\alpha \text{ in degrees})$$

**Inverse Functions:**

$\sin^{-1} x$ is the angle in $[-90^\circ, 90^\circ]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is *x*.

$\cos^{-1} x$ is the angle in $[0^\circ, 180^\circ]$ or $[0, \pi]$ whose cosine is *x*.

$\tan^{-1} x$ is the angle in $[-90^\circ, 90^\circ]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose tangent is *x*.

$\csc^{-1} x$ is the angle in $[-90^\circ, 90^\circ]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose cosecant is *x*.

$\sec^{-1} x$ is the angle in $[0^\circ, 180^\circ]$ or $[0, \pi]$ whose secant is *x*.

$\cot^{-1} x$ is the angle in $[0^\circ, 180^\circ]$ or $[0, \pi]$ whose cotangent is *x*.

**The Law of Sines:**

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

**The Law of Cosines:**

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

**Area of a Triangle:**

$$A = \tfrac{1}{2} bc\sin\alpha$$

$$A = \tfrac{1}{2} ac\sin\beta$$

$$A = \tfrac{1}{2} ab\sin\gamma$$

**Heron's Formula:**

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \text{ where } S = \frac{a+b+c}{2}$$

**Component Form of a Vector with Magnitude $r$ and Direction Angle $\theta$:**

$$\langle r\cos\theta,\ r\sin\theta \rangle$$

**Magnitude and Direction Angle of a Vector $\langle x, y \rangle$:**

$$|\langle x, y \rangle| = r = \sqrt{x^2 + y^2},\ \sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r},\ \tan\theta = \frac{y}{x}$$

**Dot Product of Two Vectors:**

$$\mathbf{A} \cdot \mathbf{B} = \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$$

**Angle Between Two Vectors:**

$$\cos\alpha = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

**Absolute Value or Modulus of a Complex Number:**

$$|a+bi| = \sqrt{a^2+b^2}$$

**Trigonometric Form of a Complex Number $z = a + bi$:**

$$z = r(\cos\theta + i\sin\theta), \text{ where}$$

$$r = \sqrt{a^2+b^2}, \text{ and } \sin\theta = \frac{b}{r},\ \cos\theta = \frac{a}{r},\ \tan\theta = \frac{b}{a}$$

**Product and Quotient of Complex Numbers:**

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right] \text{ and}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right].$$

**De Moivre's Theorem:**

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and $n$ is any positive integer, then

$$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right].$$

***n*th Roots of a Complex Number:**

For any positive integer $n$, the complex number $z = r(\cos\theta + i\sin\theta)$

has exactly $n$ distinct $n$th roots given by:

$$r^{1/n}\left[\cos\alpha + i\sin\alpha\right], \text{ where } \alpha = \frac{\theta + 360°k}{n} \text{ or } \alpha = \frac{\theta + 2k\pi}{n}, \text{ for } k = 0, 1, 2, \ldots, n-1.$$

**Polar Coordinates to Rectangular Coordinates:**

To convert $(r,\theta)$ to rectangular coordinates $(x,y)$, use $x = r\cos\theta$ and $y = r\sin\theta$.

**Rectangular Coordinates to Polar Coordinates:**

To convert $(x,y)$ to polar coordinates $(r,\theta)$, use $r = \sqrt{x^2+y^2}$ and $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$.

**Converting Equations Between Rectangular and Polar:**

Use $x^2 + y^2 = r^2$, $x = r\cos\theta$, and $y = r\sin\theta$.