10.2

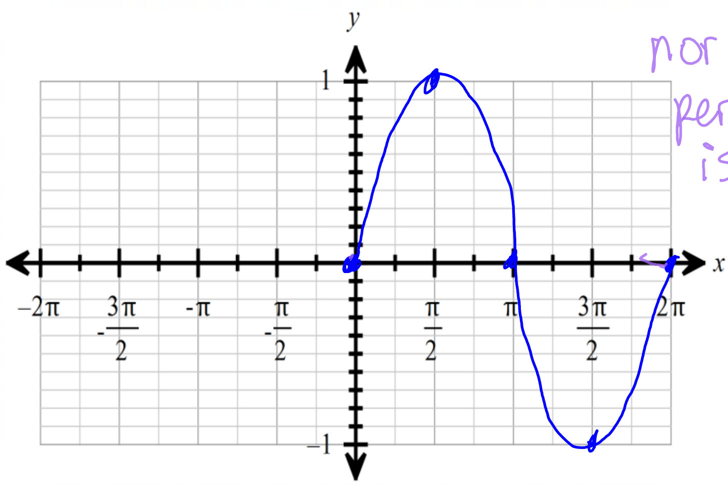
Date: 4/9/24

Objective: I can graph period and phase shift on sine and cosine functions.

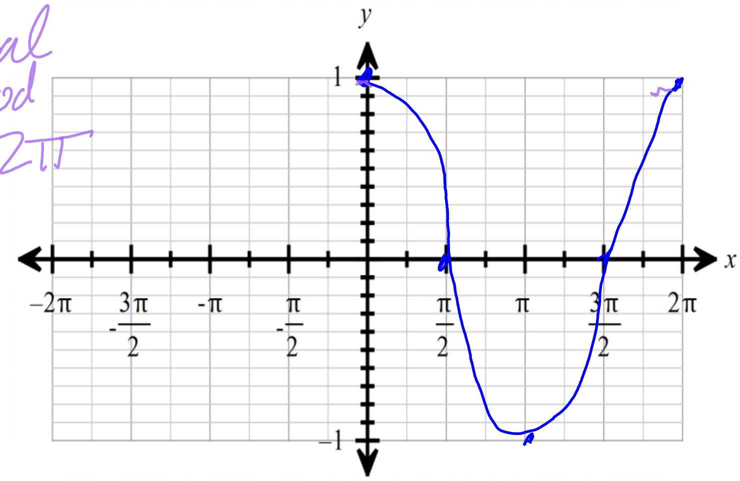
A. Graph Sine and Cosine

Parent sine graph $f(\theta) = \sin\theta$
Draw the graph and make a table.

Parent cosine graph $f(\theta) = \cos\theta$
Draw the graph and make a table.



normal period is 2π



θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin\theta$	0	1	0	−1	0

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$	1	0	−1	0	1

B. Transformations

1. From the 4 transformations in ~~9.1~~ 10.1, today we are discussing horizontal dilation and horizontal shift

2. What is the general equation for a trigonometric function? $y = \sin(b(x-c))$

Phase shift and Period:

c = **Phase Shift =** horizontal shift ⟷

add opp of c value to x

b = **Period=** how long it takes to complete a cycle or wave (horizontal dilation)

use b value to find period

$\text{Per} = \frac{2\pi}{b}$

3. Which variable in the equation is related to a *horizontal shift*? c
   - In the parent graph this is: 0

4. Which variable in the equation is related to a *horizontal stretch*? b
   - In the parent graph this is: 1
   - This is used to find the period. The formula for *period* is: $\frac{2\pi}{b}$
   - The *period* in the parent graph is 2π

$\text{Per} = \frac{2\pi}{3}$

5. *Frequency* is defined as the number of oscillations or rotations per unit of time. $\frac{b}{2\pi}$
   - *Frequency* is the reciprocal of the period. The formula for *frequency* is $\frac{b}{2\pi}$
   - The *frequency* in the parent graph is $\frac{1}{2\pi}$

$\text{freq.} = \frac{3}{2\pi}$

D. Making the Graph. (Phase Shift and Period)

EX. 1) $f(\theta) = \sin(\theta + \pi)$ c: π

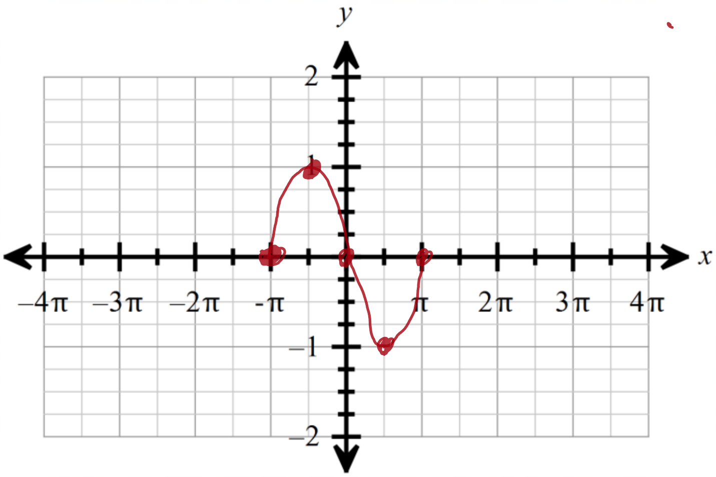
Phase Shift: left π b: 1 Period: 2π Freq.: $\frac{1}{2\pi}$

$\frac{2\pi}{1}$

Transformations: translate left π

$0-\pi$ $\frac{\pi}{2}-\pi$ $\pi-\pi$ $2\pi-\pi$

$\theta - \pi$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin\theta$	0	1	0	-1	0



EX. 2) $f(\theta) = \cos\left(\theta - \frac{\pi}{4}\right)$ c: $\frac{\pi}{4}$

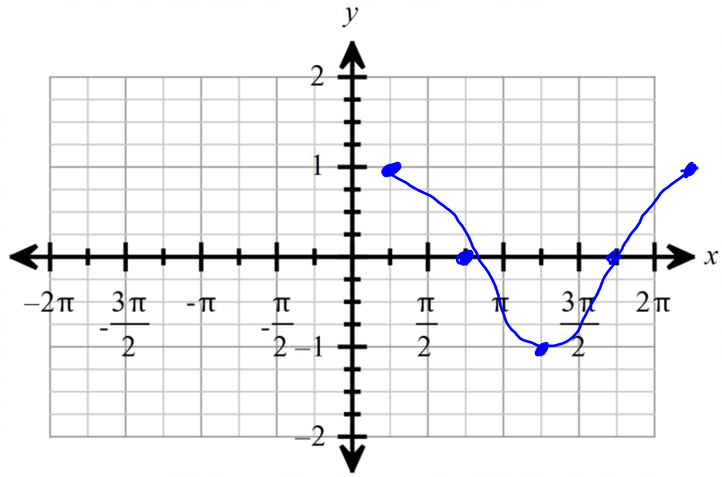
Phase Shift: right $\frac{\pi}{4}$ b: 1 Period: 2π Freq.: $\frac{1}{2\pi}$

$\frac{2\pi}{1}$

Transformations: translate right $\frac{\pi}{4}$

$0+\frac{\pi}{4}$ $\frac{\pi}{2}+\frac{\pi}{4}$ $\pi+\frac{\pi}{4}$ $2\pi+\frac{\pi}{4}$

$\theta + \frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$	1	0	-1	0	1



EX. 3) $f(\theta) = \cos(2\theta)$ c: 0

Phase Shift: 0 b: 2 Period: π Freq.: $\frac{1}{\pi}$

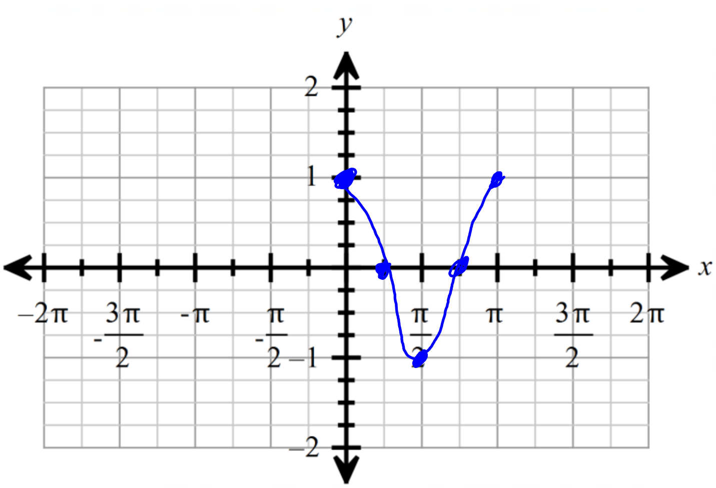
$\frac{2\pi}{2}$

Transformations: horizontal stretch of $\frac{1}{2}$

OR Period is π

$\frac{0}{2}$ $\frac{\pi}{2}\div 2$ $\frac{\pi}{2}$ $\frac{2\pi}{2}$

$\frac{\theta}{2}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$	1	0	-1	0	1



EX. 4) $f(\theta) = \sin\left(\frac{\theta}{4}\right)$ c: 0

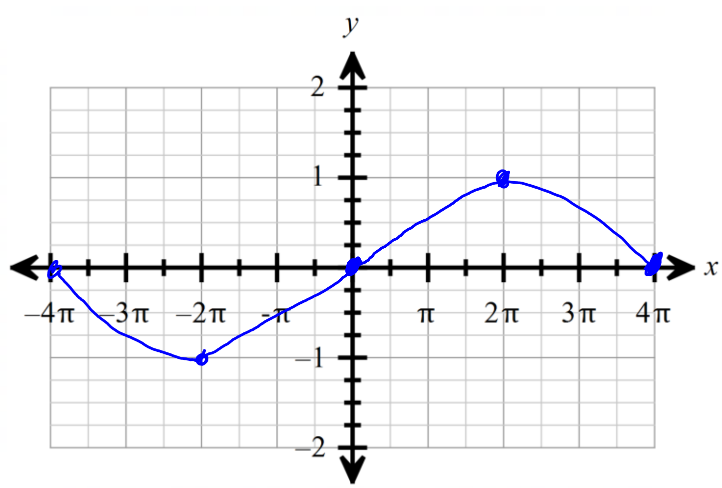
Phase Shift: 0 b: $\frac{1}{4}$ Period: 8π Freq.: $\frac{1}{8\pi}$

$\frac{2\pi}{\frac{1}{4}} = 4 \cdot 2\pi$

Transformations: hori stretch of 4

$4 \cdot 0$ 4π $4 \cdot 2\pi$

4θ	0	2π	4π	6π	8π
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin\theta$	0	1	0	-1	0



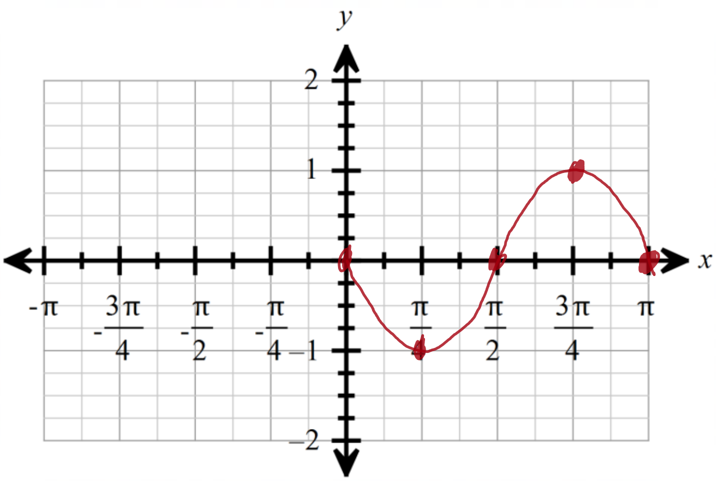
EX. 5) $f(\theta) = -\sin(2(\theta - \pi))$ c: π

Phase Shift: right π b: 2 Period: π ($\frac{2\pi}{2}$) Freq.: $\frac{1}{\pi}$

Transformations: reflect over x-axis
hor stretch of $\frac{1}{2}$
translate right π

$\frac{\theta}{2}+\pi$

$\frac{\theta}{2}+\pi$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin\theta$	0	1	0	-1	0
$-y$	0	-1	0	1	0



EX. 6) $f(\theta) = \cos 3\left(\theta + \frac{\pi}{3}\right)$ c: $\frac{-\pi}{3}$

Phase Shift: left $\frac{\pi}{3}$ b: 3 Period: $\frac{2\pi}{3}$ Freq.: $\frac{3\pi}{2}$

Transformations: hor stretch of $\frac{1}{3}$
translate left $\frac{\pi}{3}$

$\frac{\theta}{3}-\frac{\pi}{3}$	$-\frac{\pi}{3}$ ($\frac{0}{3}-\frac{\pi}{3}$)	$-\frac{\pi}{6}$	0 ($\frac{\pi}{3}-\frac{\pi}{3}$)	$\frac{\pi}{6}$	$\frac{\pi}{3}$ ($\frac{2\pi}{3}-\frac{\pi}{3}$)
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$	1	0	-1	0	1

counting by 6's so 2 tick marks = $\frac{\pi}{3}$